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A MODEL OF MEDICAL RESEARCH RESOURCE ALLOCATION

In this paper we attempt to analyze the following problem: If it were possible to buy partial or complete immunity from a particular disease for a specified time period (perhaps infinite), how much would such an option be worth? In particular, suppose that P_d is the probability of contracting the disease if no action is taken, but an individual may spend money to reduce the probability to bP_d, where $0 \leq b \leq 1$. If $b = 1$, no action is taken; if $b = 0$, there is complete immunization. This problem is a simplified version of the public policy problem of determining the allocation of funds to medical research, since much of that research is aimed at reducing the probability of contracting various diseases. The problem is simplified in that it is assumed that the cost and expected results of the research can be known in advance.

In our model, utility is assumed to be a function of two variables -- wealth, x, and health status, which is reflected by the probability, P, of contracting the disease. For simplicity, we assume that a healthy individual's utility is proportional to x.[1] If the individual contracts the disease, we will assume his utility is reduced to (1-a)x, where a, the discomfort, is a number between zero and one. Unless anxiety is important,[2] a lottery argument shows that for fixed x_0, $U(x_0,P)$ is linear in P.[3] Thus,

$$(1) \qquad U(x,P) = (1-P)U(x,0) + (1-a)PU(x,0) = (1-aP)x \ .$$

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[1] This assumption is not unreasonable, since expenditures on health research are small relative to total wealth. By a linear transformation, we can assume $U(x) = x$.

[2] We assume that the individual is not particularly susceptible to the disease, so that his probability of contracting it is low.

[3] We assume an individual is indifferent between, say, a 0.25 probability of death, and a lottery whose prizes are (with equal probability) a 0.4 probability of death and a 0.1 probability of death. For this to be true, utility must be linear in P. Raiffa gives an interesting discussion of the problem of assigning such utilities where the "disease" is death (a=1). Howard Raiffa, Preferences for Multi-Attributed Alternatives, The RAND Corporation, RM-5868-DOT/RC, April 1969, pp. 81-93.

Let the initial wealth be w, and the research expenditure r. Then $P = b(r)P_d$, and our problem is finding r to maximize $U = (1-ab(r)P_d)(w-r)$.

$$\frac{dU}{dr} = -(1-ab(r)P_d) - ab'(r)P_d(w-r) . \tag{2}$$

This expression is zero whenever

$$b(r) - b'(r)(w-r) = \frac{1}{aP_d} . \tag{3}$$

Initially assume that b(r) has the form shown in Figure 1, e.e., that $b(0) = 1$, $b(r) \geq 0$, $b'(r) < 0$, and $b''(r)$ 0. This allows for possible diminishing returns to research expenditures, but not for set-up costs.

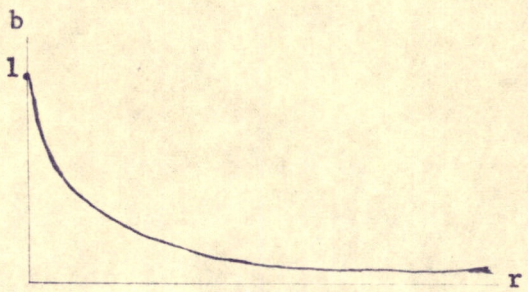

Figure 1.

Although reductions of the incidence of disease are abrupt (as with the discovery of the polio vaccine), we assume that b represents the expected reduction and hence is a smooth function of r. Since utility is assumed to be linear in money and disease, there are no problems of risk aversion.

With these assumptions on b, note that

$$\frac{d}{dr}[b(r) - b'(r)(w-r)] = 2b'(r) - b''(r)(w-r) < 0 . \tag{4}$$

Thus (3) will be satisfied by at most one value of r for any choice

of P_d and a. If $P_d = 0$ (the disease never occurs) or if $a = 0$ (the disease causes no discomfort) then $\frac{1}{aP_d}$ will be infinite and (3) is never satisfied. Thus r should be zero. If the disease does have unpleasant effects, and if the cost of research is not too high, i.e., if $b'(0)$ is not close to zero, then (3) will be satisfied for some $r > 0$.

Let us see how research expenditures vary with a, P_d, w, and b. If P_d, the initial incidence, decreases, then $\frac{1}{P_d a}$ increases and by (4), r must decrease. Similarly if a decreases [the discomfort associated with disease decreases] then r must decrease. If w increases, $b(r) - b'(r)(w-r)$ increases, and so r must increase to maintain equality in (3).

Finally, if it costs αr, instead of r, to get a reduction b(r), $\frac{dU}{dr}$ is zero when

$$b(r) - b'(r)((w/\alpha)-r) = 1/aP_d \quad . \tag{5}$$

Therefore, the amount of immunity purchased, b, is the same as that purchased with the original research costs with the initial wealth divided by α. Thus, if the research cost factor, α, increases, b decreases. The total research expenditure αr may increase or decrease depending on the shape of b(r), and the initial wealth w.

THE EFFECT OF AGE

Frederick Pohl, in Search the Sky, describes a world in which, as a result of too much geriatric research, old codgers have all the fun and power and young people are drudges. Would our model lead to this result? In other words, from the viewpoint of our model, who benefits more when the incidence of a disease is reduced -- a young person who is otherwise healthy or an old person who has an independent chance D of dying?

The young person would have the utility function given by (1), but for the old person,

(6) $$\pi(x,P,D) = (1-D)(1-aP)\,\pi(x,0) \ .$$

Thus 1 - D is just a scale factor, and the old man is willing to spend just as much as the youth to have the disease prevented even though it only helps him when he lives.[1]

THE REDUCTION IN CONSEQUENCES

For simplicity, we have assumed that only the incidence of the disease can be altered by research. In fact, much research is concerned with ameliorating its consequences.[2] Research on cures or therapies means that a, the discomfort, becomes a function of r.

Equation (1) is symmetric in a and P so if P is fixed, the earlier analysis can be trivially modified. If both a and P are controllable, we have to make the technical decision of minimizing $a(r_1) \cdot P(r_2)$ subject to $r_1 + r_2 = r$, and then determine the optimal r.[3]

A CONSTANT RETURNS WORLD

In this section, we consider the case of linear research costs. We shall then be able to compute the second order effects on b and r of varying P_d, a, and w.[4] Despite the linearity of the utility function and of b(r), decreasing marginal compensations for changes in the parameters appear.

Let T be the price of total immunity, so that b(r) has the form of Figure 2.

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[1] Since the old man can't take it with him, D acts as a lump-sum tax on his utility. Note also that a person should pay more to reduce his chance of dying from 60 percent to 59 percent than from 2 percent to 0 percent. In the first case his percentage chance of living goes from 40 to 41 and in the second from 98 to 100. It is the ratios 40/41 and 98/100 which are critical. See Raiffa, op. cit. for a more complete discussion.

[2] We wonder if this means that the price of "prevention" is more than sixteen times the price of "cure."

[3] r_1 is that part of the research budget spent on cures and therapies, and r_2 is that part spent on prevention.

[4] These could be computed, more generally, by assuming $b'''(r)$ small.

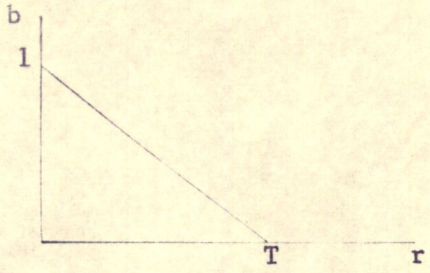

Figure .2.

(7) $$b(r) = 1 - \frac{r}{T} \qquad 0 \leq r \leq t \ ;$$

for $r < 0$, $b = 1$; for $r > T$, $b = 0$.

Equation (3) becomes

(8) $$(1-\frac{r}{T}) + \frac{1}{T}(w-r) = \frac{1}{aP_d} \ .$$

Solving for r, we get

(9) $$r = \frac{1}{2}(w-T(\frac{1}{aP_d} - 1)) \qquad 0 \leq r \leq T.$$

If the right hand side (r.h.s.) ≤ 0, then $r = 0$; if the r.h.s. $\geq T$, $r = T$. To get a demand curve for immunity, $1 - b$, we substitute to get

(10) $$1 - b = \frac{r}{T} = \frac{1}{2}\left(\frac{w}{T} + 1 - \frac{1}{aP_d}\right) \qquad 0 \leq b \leq 1.$$

The demand for immunity as a function of the price T is thus a truncated hyperbola, as shown in Figure 3. The analysis in the first section of the paper gives us the signs of the first derivatives of (10) (which differentiation of (10) confirms), but the second order properties of this model are interesting:

(11) $$\frac{d^2(1-b)}{dP_d^2} = \frac{1}{aP_d^3} < 0 \ .$$

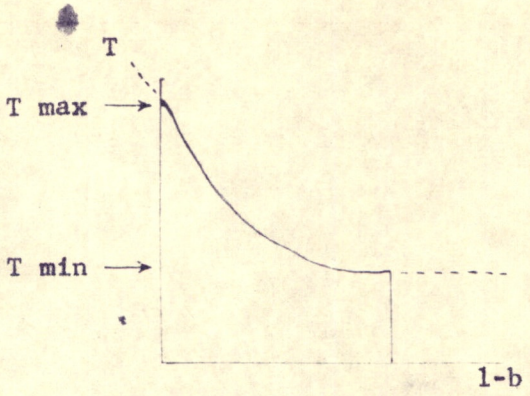

Figure 3.

Since P_d and a are positive, the second derivative is negative. As the probability of contracting the disease rises, we are willing to buy more protection against the disease, but we buy proportionately less protection. This is the same type of behavior that would be exhibited by an individual with a convex utility function in an ordinary decision-making under uncertainty problem, as shown in Figure 4:

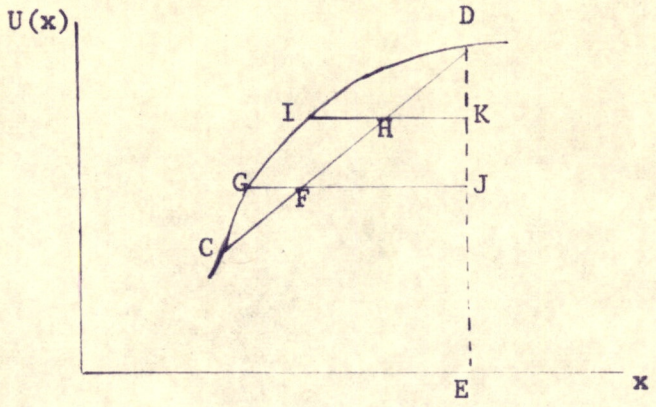

Figure 4.

In this diagram CGID is the individual's utility function; there are two states of the world possible: C and D. The individual is currently in state D. The chord connecting C and D can be used to find the expected utility of any risky prospect which has the outcomes C and D. Lines such as IHK and GFJ show the maximum amount the individual would spend of his current wealth D to avoid prospect C, given that the odds of obtaining C are such that expected utility is at IHK

or GFJ. As the odds of obtaining C rise, the individual is willing to spend more to avoid C, but the increment he spends to avoid C falls.

$$(12) \qquad d^2(1-b)/da^2 = -\frac{1}{a^3 P_d} < 0 .$$

As discomfort rises, the individual spends proportionately less to avoid increasing it. This can be seen in Figure 4 by holding the probability of incurring the disease constant at, say, 0.5. Then the amount the individual is willing to pay is given by the length of a line parallel to the X-axis from the midpoint of the chord connecting C and D to ED. As C falls toward the origin (a increasing), the length of the line will increase, but less than proportionately.

$$(13) \qquad \frac{d^2(1-b)}{dw^2} = 0 .$$

As the amount of resources available to spend rises, the consumer is willing to buy more protection against the disease, but he spends the same proportion of any increment to his resources to reduce the probability of disease. If $r = 0$ when $w = 0$, this implies that medical research is neither a luxury nor a necessity. How realistic this result is must be left for the reader to judge.[1]

$$(14) \qquad d(1-b)/dT = -0.5w/T^2 < 0.$$

$$(15) \qquad d^2(1-b)/dT^2 = w/T^3 > 0 .$$

As the price of immunization rises, we buy less of it, but an equal (absolute) price rise at a higher price causes less of a reduction than the same price rise at a lower price. (This should be obvious

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[1] Other studies have found the income elasticity of expenditures on medical care to be near 1.0. See Paul J. Feldstein and W. John Carr, "The Effect of Income on Medical Care Spending," Proceedings, American Statistical Association, 1964, pp. 93-105.

from Figure 3.) If we write the demand curve in the form:

$$1 - b = j + (kw/T), \qquad j,k > 0 , \tag{16}$$

we find that price elasticity of demand E equals:

$$E = -kw/(jT+kw). \tag{17}$$

Further,

$$dE/dy = -jkT/(jT+kw)^2 < 0 . \tag{18}$$

As wealth rises, the consumer becomes less sensitive to variations in price. The signs of these derivatives accord with intuition, so that, simple as it is, the model may be a reasonable approximation to reality.

A MULTI-DISEASE WORLD

The model can easily solve the problem of how to allocate funds for research into several diseases, if we assume the chance of contracting two diseases is small enough to be ignored and continue to make the assumption that the research cost on each disease is linear ($b'' = 0$). Let

P_i be the current probability of contracting disease i,

b_i the expected degree of immunity purchased from disease i (when $b = 0$ there is complete immunity),

T_i the price of complete immunity,

a_i the discomfort associated with the disease.

Thus $(1-b_i)T_i = r_i$, the money spent on i^{th} disease.

$$U = [1 - \sum b_i P_i a_i] (w - \sum r_i) . \tag{19}$$

Since $(1-b_i)T_i = r_i$,

$$\frac{\partial b_i}{\partial r_i} = - \frac{1}{T_i} ,$$

and so

$$(20) \qquad \frac{\partial U}{\partial r_i} = -\left[1 - \sum b_j P_j a_j\right] + \left(w - \sum r_j\right) \frac{P_i a_i}{T_i} .$$

The only term in the formula for $\frac{\partial U}{\partial r_i}$ which varies with i is $\frac{P_i a_i}{T_i}$, the relief per dollar of research funds. Thus $\frac{P_i a_i}{T_i}$ is the critical factor in deciding which disease to invest research resources in. Diseases should be completely prevented, in order of decreasing relief, until for some i, $\left[1 - \sum b_j P_j a_j\right] = \left(w - \sum r_j\right) \frac{P_i a_i}{T_i}$ or until a budget constraint is reached.

This concentration of resources is the result of linear research costs and the linear value to an individual of reductions in P. A funding organization, such as the National Institute of Health, might feel that it should get at least some minimal level of results. Since the returns $b_i(r_i)$ are not certain, but are expected returns, it might well want to invest in "inferior" diseases (with lower $P_i a_i / T_i$), just as a risk-averse investor may invest in stocks with less than the optimal expected rate of return to bring down his variance.

AGGREGATION AND AN ILLUSTRATIVE APPLICATION

To this point the analysis has treated the problem of allocation of resources to medical research as though research were an ordinary private good. Of course, it is not; the opportunity cost of an additional person using the knowledge produced by the research is zero. Further, without a patent system (or something similar) it may be difficult for individuals to capture the benefits of resources expended in research. For both these reasons biomedical research is heavily supported by the government.

Does our model give any insights which are relevant for public policy? We believe it does. Since an additional person may use the knowledge produced at zero cost to society, the total benefit of a

given research finding is the sum of the benefits it yields to any individual. We thus suggest the following as a theoretically correct criterion for resource allocation (ignoring distributional considerations): proceed to where the marginal benefit from additional resources allocated to research (as defined above) equals the marginal opportunity cost of those resources.[1] Due allowance may be made for uncertainty in the benefit or cost streams in any of a number of ways. We do not discuss the uncertainty problem.

Notice that this criterion avoids two errors of the approach which values the research output as the increment it makes to a discounted GNP stream: 1) Consumption benefits, or what Schelling calls the value of "life" as opposed to "livelihood,"[2] are included in the analysis; 2) externalities, or the value the individual places on reductions in the probability of death for others, are also included.

There would be two problems in attempting to apply this model empirically: 1) Since financing takes place through the existing tax structure, the cost to a particular individual may exceed the benefits he receives. If side payments (generally in the form of other legislation) are not permitted, there will be redistributive effects. These may be desirable or undesirable. If undesirable, they may be reason for not undertaking the research project. We would hazard the guess that the redistributive effects of research programs across income groups are rather small; higher income groups would probably benefit more but they would also pay more; 2) there is a problem in actually eliciting individual preferences.[3] Ideally, we could ask individuals: if for a given price they could reduce their chance of contracting a certain disease by a given number of percentage points, how much protection they would buy. P_d and a probably vary by income-wealth and

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[1] If markets are not elsewhere perfect, there is a second best problem which we ignore.

[2] Thomas C. Schelling, "The Life You Save May Be Your Own," in Samuel B. Chase, Jr., ed., Problems in Public Expenditures Analysis, Washington, The Brookings Institution, 1968.

[3] See Raiffa, op. cit., for some "tricks of the trade."

age classes, and P_d may also vary with other characteristics such as smoking habits. Further, the proportion of the present value of his lifetime income stream plus his present wealth which an individual is willing to spend in the present period may also vary with income-wealth and age. For these reasons one would need to sample various income-wealth and age classes to estimate the amount of protection any given class would buy. Then, by summing the resulting estimates, one could estimate the total value to the population of reducing the probability of contracting a given disease. This could then be compared with the production function for medical research on that disease to ascertain the amount of research at which the marginal benefit equalled the marginal cost.

In lieu of performing such an experiment, we have for illustrative purposes taken over the model directly and used it to estimate what it might be worth in the aggregate to consumers to reduce the probability of death by 1 percent. To do this we have taken P_d to be 0.01.[1] A 1 percent reduction in the probability of death means b = 0.99. The wealth available to spend on medical research is assumed to be Net National Product plus existing wealth. Net National Product in 1965 was $625.1 billion (1965 dollars).[2] Wealth in 1966 was $2.46 trillion (1966 dollars).[3] We have used $2.4 trillion as an estimate of wealth in 1965, but an error in this figure is not critical.[4] Since the reduction in the probability of death is taken to be in perpetuity, we take existing wealth plus the present value of Net National Product in perpetuity to be equal to the wealth variable x in equation 10. (In effect, society borrows to spend money in the present year on medical research. For the small sum we are considering, the assumption

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[1] The actual 1965 death rate was 0.0094. (United States Bureau of the Census, Statistical Abstract, 1967, Washington: GPO, 1967, p. 59.)

[2] Council of Economic Advisers, Economic Report of the President, 1969, Washington: GPO, 1969, Table B-13.

[3] Statistical Abstract, 1967, p. 344.

[4] The next earlier year for which statistics are available is 1958. Then wealth was 1.7 trillion (1958 dollars). Ibid.

of a constant discount rate seems reasonable.) In Table 1 we show the estimate yielded by equation 10 of the maximum amount consumers would be willing to pay at various discount rates and various values of a to reduce the probability of death 1 percent. Since we are considering death, the most reasonable value of a is one. The discount rates should be adjusted for exogenous technical progress; if this is assumed to be approximately 2 percent annually, the discount rate should be approximately 2 percent below what it otherwise would be. The discount rates used in Table 1 are 2 percent, 5 percent, and 10 percent, reflecting rates of time preference of 4 percent, 7 percent, and 12 percent.

Table 1

MAXIMUM VALUE TO CONSUMERS OF A 1 PERCENT REDUCTION IN THE PROBABILITY OF DEATH (in billions of 1965 dollars)

a	r (discount rate)		
	0.02	0.05	0.10
1	3.4	1.5	0.9
0.9	3.1	1.3	0.8
0.5	1.7	0.8	0.5

Actual expenditures for medical research in 1965 were $1.5 billion.[1] (This excludes any expenditures made by corporations. $1.2 of the $1.5 billion came from the federal government; the remainder came from state and local government and philanthropy.) Note, however, that many expenditures which reduce the probability of death (e.g., better highways) are not part of medical research, and that the $1.5 billion in medical research also reduced the probability and consequences of sickness. Accepting these qualifications, the question which remains for policy is whether the expected benefit of these expenditures (net of any allowance for risk) exceeds, equals, or falls

[1] Dorothy P. Rice and Barbara J. Cooper, "National Health Expenditures, 1950-1967," Social Security Bulletin, 32:1, January 1969, p. 4.

short of a 1 percent reduction in the probability of death. Since for the entire decade 1955 to 1965 the change in life expectancy was only 1 percent (69.5 years to 70.2 years), and the change in the age adjusted death rate was only 4 percent (7.7 per 1000 to 7.4), there is some reason to believe that the expected benefit may be less than 1 percent.[2]

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[1]United States Bureau of the Census, Statistical Abstract, 1967, op. cit., pp. 53, 56; ibid., Historical Statistics of the United States, Washington: GPO, 1960, p. 27. Research expenditures totalled $5.8 billion between 1960 and 1965 (inclusive). Ida C. Merriam, "Social Welfare Expenditures, 1964-1965," Social Security Bulletin, October 1965, p. 4.